The Central Limit Theorem for Means

Lecture 31
Section 8.4

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Outline

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Experimental Results

Samp. Mean	Frequency
90	1
96	1
102	0
108	0
114	3
120	4
126	14
132	12
138	10
144	13
150	12
156	5
162	17
168	3
174	5

The distribution of sample means

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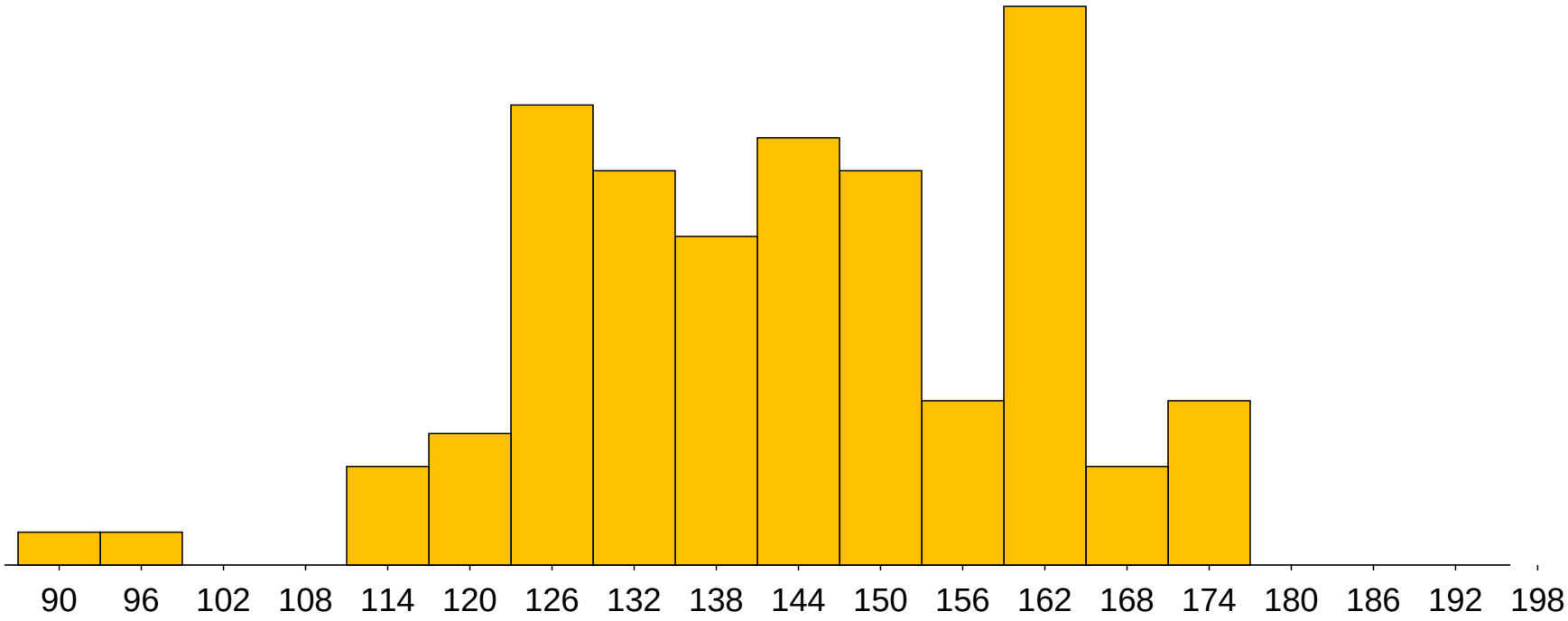
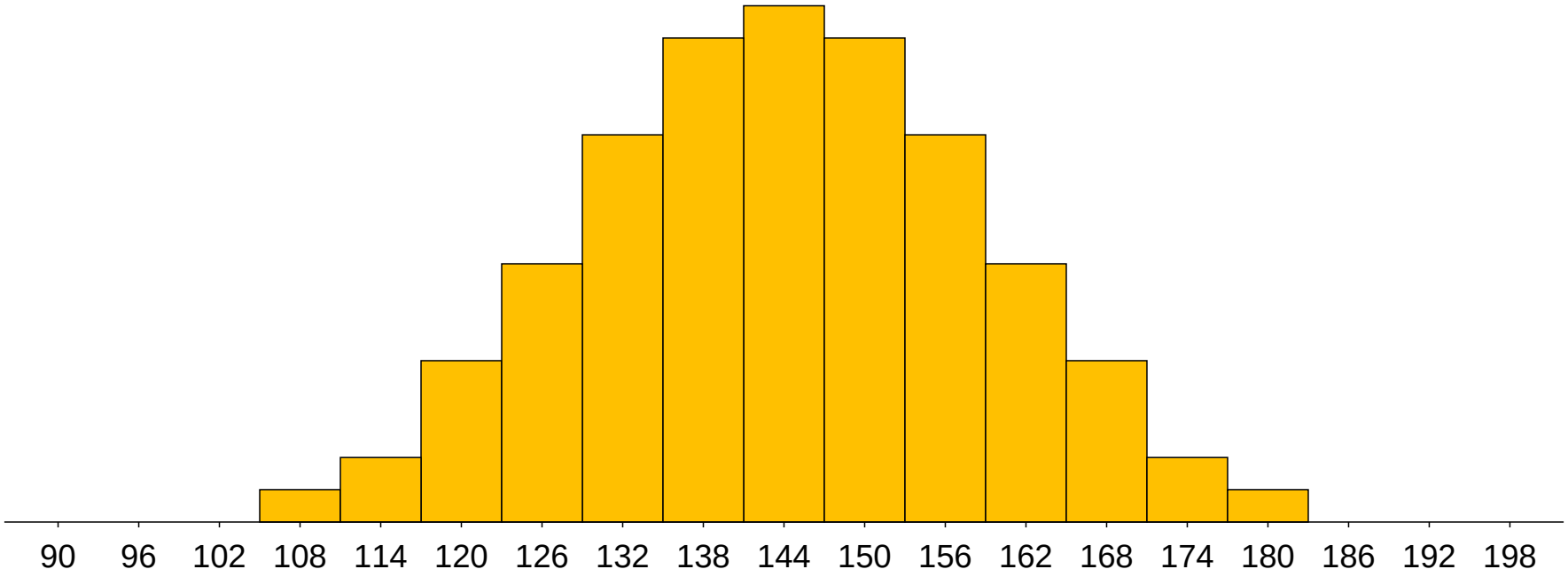
- What was the standard deviation of $\overline{x}$ in our experiment?
- It turned out to be 17.31 (vs. 15.17).

Experimental Results



Our results

Experimental Results



Theoretical results

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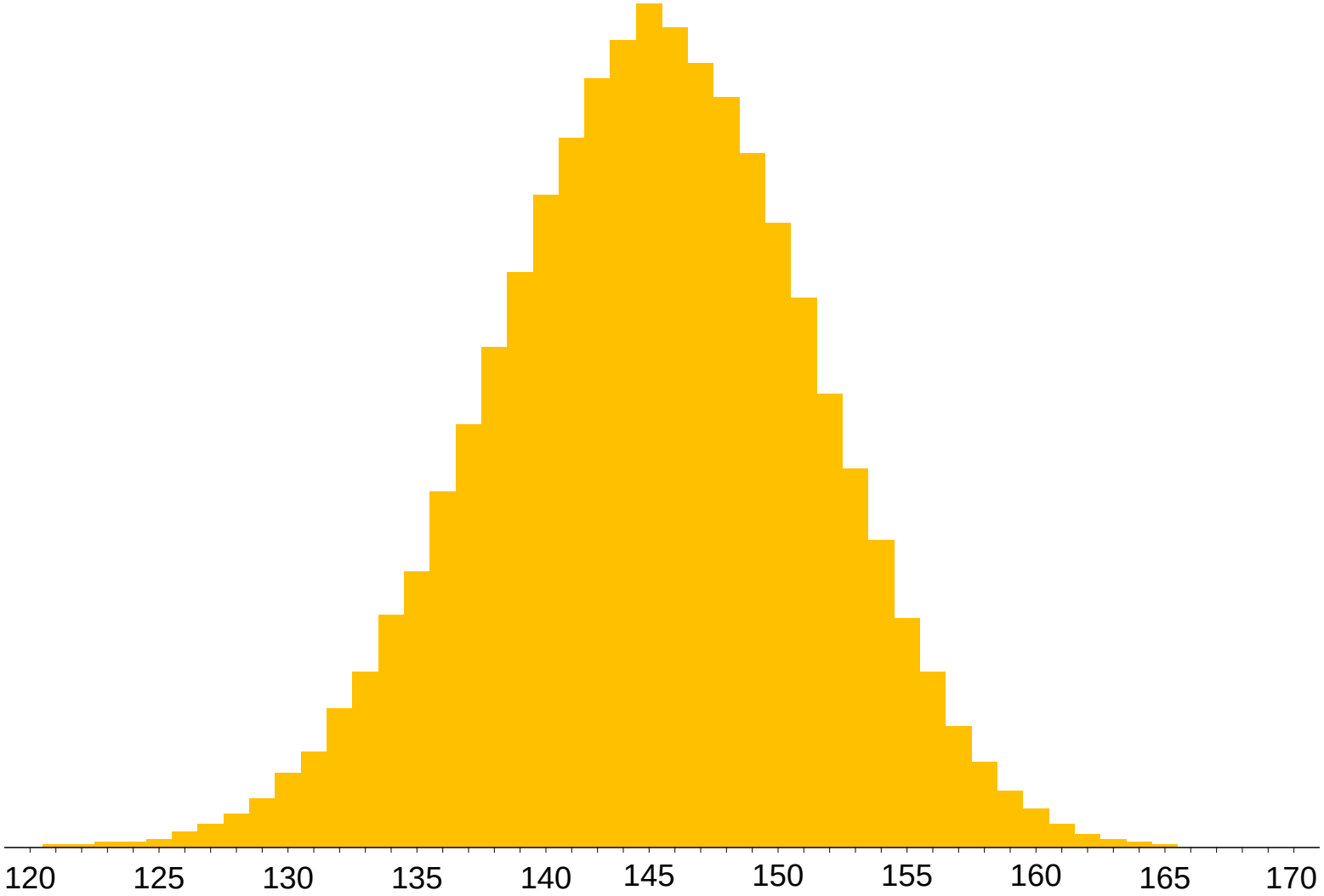
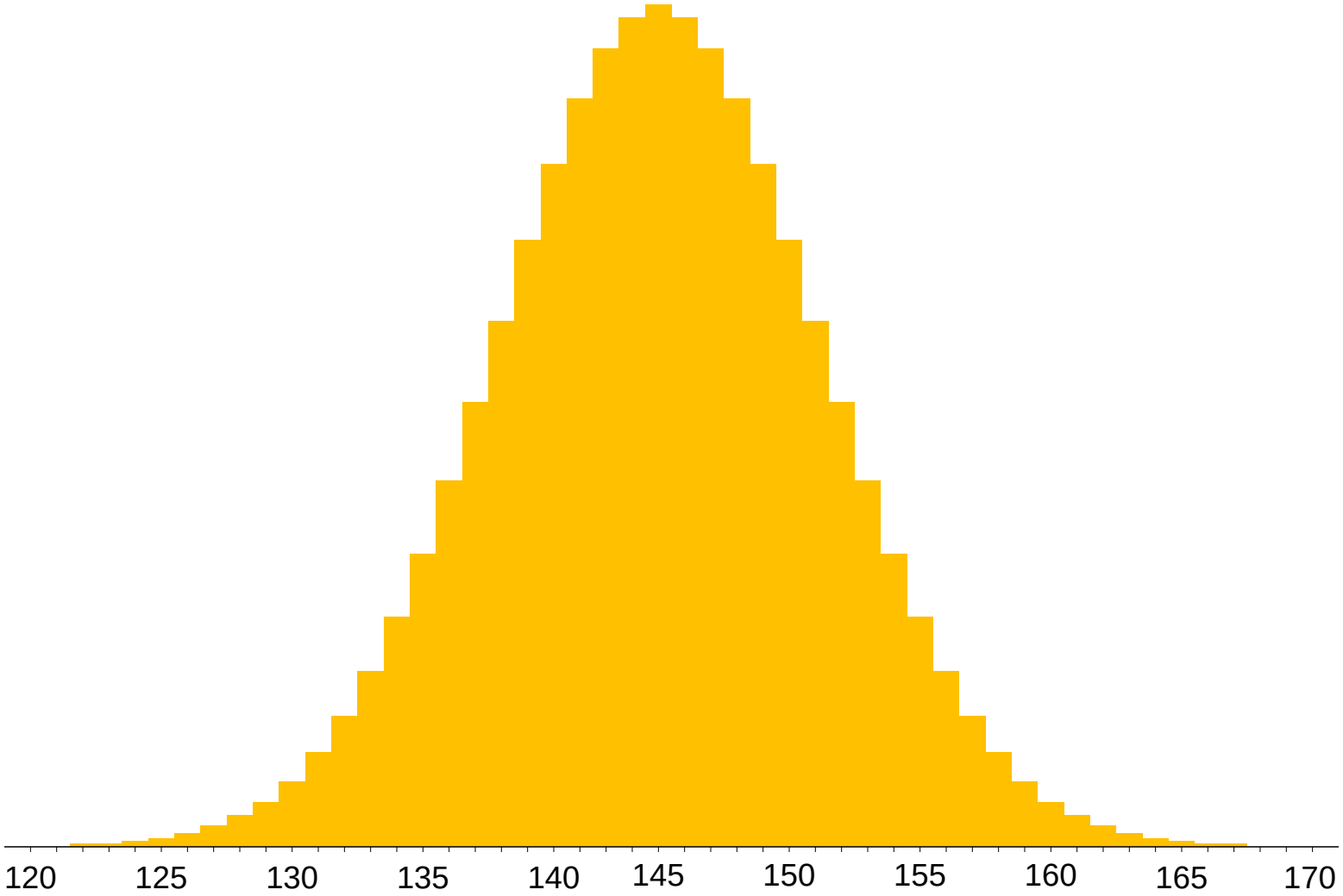
- I took 100,000 random samples, each of size 60.

Experimental Results



My results

Experimental Results



Theoretical results

Experimental Results

- The average value of my 100,000 $\overline{x}$'s was 143.997 (vs. 144).
- The standard deviation of my 100,000 $\overline{x}$'s was 6.206 (vs. 6.197).

Outline

The Central Limit Theorem for Means

Theorem (Central Limit Theorem for Means (full version))

Let a population have mean μ and standard deviation σ. The population of sample means $\overline{x}$ from all possible samples of size n

- *Has mean $\mu_{\overline{x}} = \mu$ and*
- *Has standard deviation $\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}}$.*

Furthermore, the distribution of $\overline{x}$ is normal if either

- *The original population itself is normal, or*
- *The sample size is large (at least 30).*

Outline

Heights of Adult Males

Example (Heights of Adult Males)

The heights of adult U.S. males has a normal distribution with a mean of 69.5 inches with a standard deviation of 2.9 inches. If we select 100 males at random from this population,

- What proportion of this sample do we expect to be between 68 and 71 inches tall?
- What is the probability that the sample mean height will be at least 70 inches?

Maintenance Costs

Example (Maintenance Costs)

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- Can we predict with any accuracy the average cost of the next 100 repairs?

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- Can we predict with any accuracy the cost of the next repair?
- Can we predict with any accuracy the average cost of the next 10 repairs?
- Can we predict with any accuracy the average cost of the next 100 repairs?
- What is a reasonable upper bound on the total cost of the next 100 repairs?

Outline

Assignment

Homework

- Read Section 8.4, pages 531 - 545.
- Let's Do It! 8.8, 8.9.
- Exercises 17 - 22, 25, 28 - 30, page 551.